Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Friday 25 January 2013 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41486A

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

2.

$$\mathrm{g}(x) = \mathrm{e}^{x-1} + x - 6$$

(a) Show that the equation $\mathrm{g}(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

(2)

The root of $\mathrm{g}(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

3.

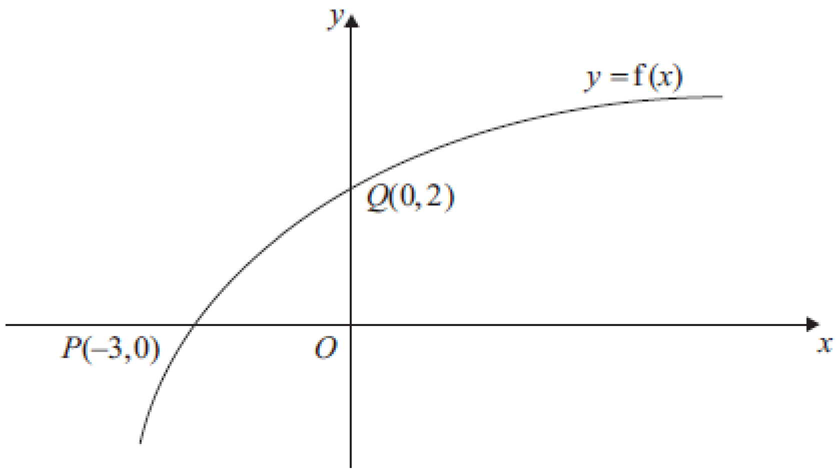


Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of ff (-3).

(2)

On separate diagrams, sketch the curve with equation

(b) $y = \mathrm{f}^{-1}(x)$,

(2)

(c) $y = \mathrm{f}(|x|) - 2$,

(2)

(d) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

4. (*a*) Express 6 cos θ + 8 sin θ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*)
$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

---

5. (i) Differentiate with respect to x

(*a*) $y = x^3 \ln 2x$,

(*b*) $y = (x + \sin 2x)^3$.

(6)

Given that $x = \cot y$,

(ii) show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-1}{1 + x^2}$.

(5)

---

6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working.

(5)

(ii) (*a*) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$k\sin^2\theta - \sin\theta = 0$, stating the value of k.

(2)

(*b*) Hence solve, for $0 \leq \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1.$$

(4)

---

7. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geq 0.$$

(*a*) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$.

(4)

(*b*) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form.

(3)

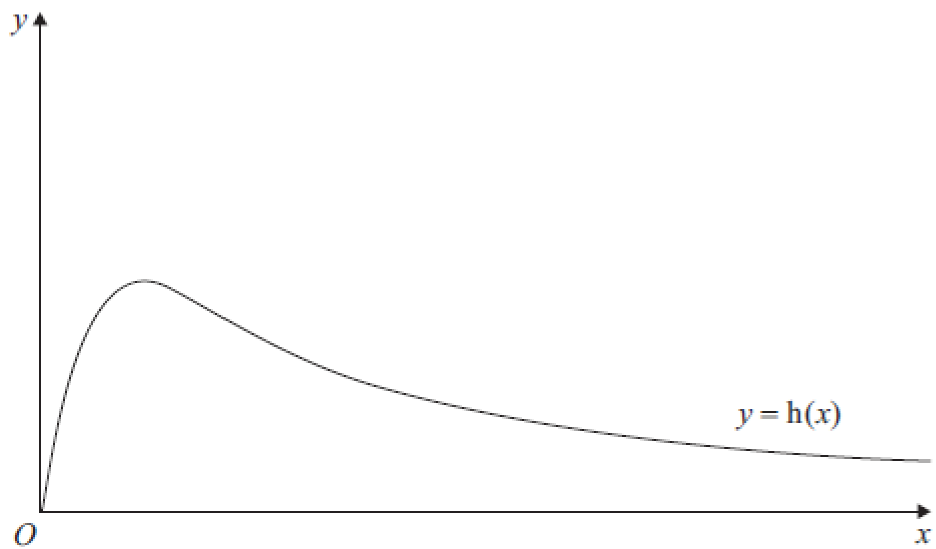


Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(*c*) Calculate the range of $\mathrm{h}(x)$.

(5)

---

8. The value of Bob's car can be calculated from the formula

$$V = 17000\mathrm{e}^{-0.25t} + 2000\mathrm{e}^{-0.5t} + 500.$$

where V is the value of the car in pounds (£) and t is the age in years.

(*a*) Find the value of the car when $t = 0$.

(1)

(*b*) Calculate the exact value of t when $V = 9500$.

(4)

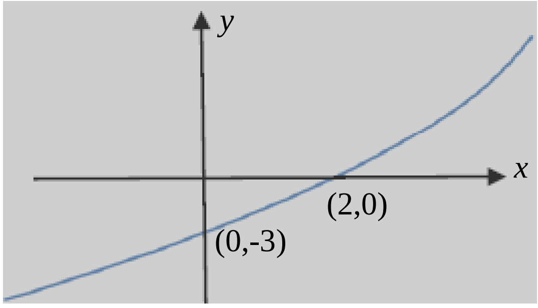
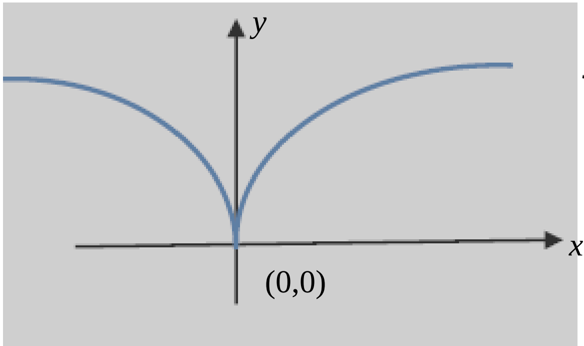
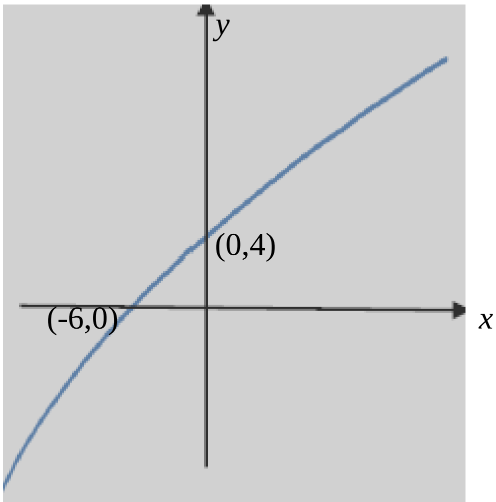
(*c*) Find the rate at which the value of the car is decreasing at the instant when $t = 8$.
Give your answer in pounds per year to the nearest pound.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)
2.	(a) $0=e^{x-1}+x-6 \Rightarrow x=\ln(6-x)+1$	M1A1*
		(2)
	(b) Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1 \Rightarrow x_1=2.3863$	M1, A1
	AWRT 4 dp. $x_2=2.2847$ $x_3=2.3125$	A1
		(3)
	(c) Chooses interval [2.3065,2.3075]	M1
	g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1
		(3)
		(8 marks)

Question Number	Scheme	Marks
3.	(a) ff(-3)= f(0),=2	M1,A1
		(2)
	(b)  $y = \mathrm{f}^{-1}(x)$	
	Shape	B1
	(0,-3) and (2,0)	B1
		(2)
	(c)  y=f(|x|)-2	
	Shape	B1
	(0,0)	B1
		(2)
	(d) 	
	Shape	B1
	(-6,0) or (0,4)	B1
	(-6,0) and (0,4)	B1
		(3)
		(9 marks)

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $\text{p}(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$\text{p}(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - \text{'}their\ \alpha\text{'} = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)
5.	(i)(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
		(3)
	(i)(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\boldsymbol{(x + \sin 2x)}^2 \times (1 + 2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\frac{\mathrm{d}x}{\mathrm{d}y} = -\text{cosec}^2 y$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ to get an expression in x	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$ cso	M1, A1*
		(5)
		(11 marks)

Question Number	Scheme	Marks
6.	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$	M1
	$(\sin 22.5+\cos 22.5)^2=1+\sin 45$	A1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ cso	A1
		(5)
	(ii) (a) $\cos 2\theta+\sin\theta=1 \Rightarrow 1-2\sin^2\theta+\sin\theta=1$	M1
	$\sin\theta-2\sin^2\theta=0$	
	$2\sin^2\theta-\sin\theta=0$ or $k=2$	A1*
		(2)
	(b) $\sin\theta(2\sin\theta-1)=0$	M1
	$\sin\theta=0,\quad \sin\theta=\frac{1}{2}$	A1
	Any two of 0,30,150,180	B1
	All four answers 0,30,150,180	A1
		(4)
		(11 marks)

Question Number	Scheme		Marks
7.	(a) $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$		M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$		M1
	$=\frac{2x}{(x^2+5)}$		A1*
			(4)
	(b) $\text{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$		M1A1
	$\text{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$	cso	A1
			(3)
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$		M1
	$\Rightarrow x=\sqrt{5}$		A1
	When $x=\sqrt{5}\Rightarrow \text{h}(x)=\frac{\sqrt{5}}{5}$		M1,A1
	Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$		A1ft
			(5) **(12 marks)**

Question Number	Scheme	Marks
8.	(a) (£) 19500	B1
		(1)
	(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	
	$17e^{-0.25t} + 2e^{-0.5t} = 9$	
	$(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4\ln(2)$ *oe*	A1
		(4)
	(c) $(\frac{\mathrm{d}V}{\mathrm{d}t}) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1
		(4)
		(9 marks)